TECHNICAL BULLETIN

UNISORB MACHINERY INSTALLATION SYSTEMS

GB-0008-2

BASIC VIBRATION AND VIBRATION ISOLATION THEORY

The object of the following is to acquaint the user with the principles and terminology encountered in analyzing basic vibratory systems. Information is presented regarding the simple vibratory system and the principles of vibration isolation plus a discussion of how these principles apply to isolated foundation design.

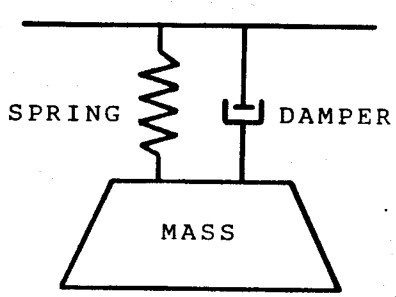
THE BASIC VIBRATORY SYSTEM

Vibratory systems are comprised of a means for storing **potential energy** (i.e., a spring) and a means for storing **Kinetic energy** (mass or inertia) and a means by which the energy is lost (dampened). The vibration of a simple vibratory system involves the alternating transfer of energy between its potential and kinetic forms. In a dampened system, some energy is dissipated at each cycle of vibration and must be replaced from an external source if a steady state vibration is to be maintained.

Although a single physical structure may store both kinetic and potential energy and may dissipate energy, the properties of that structure can be studied as separate elements.

A simple and convenient method of representing the elements of a simple vibratory system is to consider that the system is comprised of a spring, connected to a rigid infinite body and a mass which is suspended by that spring. In a classical theoretical analysis the dampening action is represented by an additional element connected in parallel with the spring which has the property of dissipating the energy without affecting the action of the spring. The mass in this model provides the method of storing kinetic energy, the spring provides the method of storing potential energy and, of course, the damper (or dash pot) provides the method dissipating the vibratory energy (see Figure 1).

Figure 1



If we displace the mass from its resting position by stretching the spring, we are imparting potential energy through the force operating against the spring. Upon releasing the spring, the potential energy stored begins to be converted to kinetic energy as the mass is accelerated from its resting position by the spring as it returns to its original length. All of the energy imparted to the system is now stored in the kinetic energy of the mass as it passes through the zero point and continues to move upward. Kinetic energy is transformed back into potential energy and once again stored in the spring. At some displacement above the at rest position the mass will come to a complete stop with the kinetic energy again having been transformed into potential energy and stored by the system. The events described are one cycle of a steady state vibration which will repeat itself indefinitely if no energy is lost from the system. In the real world, however, this would not be the case as a small amount of energy is lost during each vibratory cycle in deflecting the internal structure of the spring and through air resistance on the moving mass. This results in the displacement of the mass being slightly less with each vibration until no motion is encountered at all.

The equations defining this **"simple harmonic motion"** are:

Figure 2

2.1 Displacement $X = X_O \mathrm{SIN}\ (2\pi f T)$ — T = Time (period)

2.2 Velocity $\dot{X} = X_O(2\pi f)\cos 2\pi f T$ — f = Frequency (Hz)

2.3 Acceleration $\ddot{X} = -X_O(2\pi f)^2 \sin 2\pi f T$ — X_O = Amplitude of Displacement

The maximum values of displacement, velocity, and acceleration occur when the trigonometric function of the above are numerically equal to 1. The expressions of Figure 2 then become Figure 3.

Figure 3

3.1 Displacement $= X_O = X_O$

3.2 Velocity Max. $= \dot{X}_O = (2\pi f)X_O$

3.3 Acceleration $= \ddot{X}_O = (2\pi f)^2 X_O$

Displacement is typically expressed in inches, velocity as inches per second and acceleration as a dimensionless multiple of the gravitational acceleration g. In the English system g = 386 in/sec.2 requiring that the result obtained from equation 3.3 above be divided by 386 to obtain the traditional value.

The phrase **critical dampening** often is heard while discussing vibratory systems and vibration isolation. A condition of critical dampening is simply one in which energy is lost through the dampening device at a rate such that if the mass in the spring mass system discussed above was displaced from zero, it would return to its original position only, and the system would not be set into vibratory motion.

The term **natural frequency** of a vibratory system refers to the rate at which the vibratory motion repeats in the absence of external influence. For the simple single degree of freedom system, if the stiffness of the spring is known and the weight of the suspended mass is known, it is possible to obtain the natural frequency in cycles per second or Hz by solving the following relationship:

Figure 4

$$F_n = \frac{1}{2\pi}\sqrt{\frac{Kg}{W}}$$

Fn = natural frequency
K = spring constant
W = weight of suspended body
g = gravitational constant (386.2)
π = 3.1416

In the expression, K is the spring constant of the spring (expressed as the number of pounds required to cause the spring to be deflected one inch). The letter G refers to the universal gravitational constant, 386.2 in/sec. The letter W refers to the weight (in pounds) of the suspended mass.

This relationship can be made even simpler through a little algebra which yields the following relationship:

Figure 5

$$F_n = \frac{1}{2\pi}\sqrt{\frac{g}{d_{st}}}$$

The term d_{st} refers to the static deflection in inches of the spring element of the system ($d_{st} = \frac{Mg}{K}$) with M representing the mass and K the spring constant as before. This is the simplest and most useful form in which this relationship can be expressed for our purpose.

Looking at the relationship, it can be seen that the only variable present is the static deflection of the spring. The natural frequency of a classical vibratory system then depends directly on the amount of deflection that the spring experiences when the suspended mass is attached to it.

TRANSMISSIBILITY

Consider for a moment the force exerted on the earth by the spring. As the spring acts to decelerate and accelerate the suspended mass, a force is generated between the spring and the mass attached to it. As this force must be balanced by an equal and opposite reaction, it can then be said that the spring exerts exactly the same force on the earth as it does on the suspended mass, only in the opposite direction. It is the effect of this reaction or "transmitted" force which is our primary concern.

The ratio of the transmitted force F_t to the applied force F_o can be expressed in terms of **transmissibility T:**

Figure 6

$$\frac{F_t}{F_o} = T(SIN)(WT-\Psi)$$

where

$$T = \sqrt{\frac{1+(2\zeta W/W_n)^2}{(1-W^2/W_n^2)^2+(2\zeta W/W_n)^2}}$$

and

$$\Psi = TAN^{-1}\ \frac{2\zeta(W/W_n)^3}{1-W^2/W_n^2+4\zeta^2 W^2/W_n^2}$$

ζ = fraction of critical dampening

Ψ = phase angle between the applied and transmitted forces

W = angular applied frequency

W_n = angular natural frequency

Chart A graphically presents values of dampening coefficients for phase angles Ψ and Chart B graphically presents values of Transmissibility T for various forcing to natural frequency ratios for several dampening coefficients.

For systems where the coefficient of critical dampening is .5 or less the transmissibility of the system becomes less than 1 when the F to Fn ratio exceeds 1.414. When higher dampening is available, the transmissibility over the entire range is reduced and under maximum conditions may be held below 1 for a considerable portion of the range.

Consider next that the suspended mass is "forced" by external means to vibrate at some frequency F_o.

This external or forcing frequency vibration may be the result of internal rotating components (as in the case of a machine tool), internal reciprocating components (as in the case of a press or compressor), or may originate from a multitude of sources external to the system. In this situation, the mass will no longer vibrate at its resonant frequency (if sufficient dampening is designed into the system). Vibrations occur at the new or forcing frequency. In this situation, the force transmitted to the foundation is directly proportional to the spring deflection during the vibratory cycle. Expressing this deflection as the unknown X we obtain:

Figure 7

$$X = TF_o SIN(WT-\Psi)$$

T and Ψ are defined as in Figure 6 and are shown graphically in Charts A and B respectively.

Note that the expression for T holds true when dealing with force, displacement or acceleration.

RESONANCE

The term resonance refers to the condition which exists when an external forcing frequency occurs at the natural frequency of the vibratory system. In this particular case, referring to Chart A, it can be seen that the transmissibility is much greater than 1 and, in fact, in an undampened system approaches infinity. That is to say that the force applied to the foundation would be several times that applied to the moving mass by the external source. In actual cases, however, some dampening is present and the amplification is limited to about 4.5 to 1. At resonance the external force applied to the system continues to increase the kinetic and potential energy levels with no losses other than through dampening.

VIBRATION ISOLATION

With this understanding of some of the basic principles involved, it is now appropriate to discuss the basic design of a **vibration isolating mounting system.** The primary objective here is to segregate a particular vibratory system (machine or machine/foundation) from the remainder of its environment in such a way that the transmission of the forces through that mounting system to or from the remainder of the environment is limited.

This is most simply accomplished through careful design of the mounting system to achieve a forcing to natural frequency ratio which produces the desired relationship (Chart A) to produce isolation.

For example, consider a small non-support critical machine tool mounted on Unisorb Level-Rite mounts. The mass of the machine tool itself provides the means for the storage of kinetic energy. The felt pads in the Unisorb Level-Rite mounts serve as the "spring" of the system providing a means for the storage of potential energy. The felt pads also provide a means for the dampening out of a certain portion of the energy input by the machine proper as kinetic energy is transformed to potential energy.

The machine, its mounts, and the foundation on which they rest provide the complete vibratory system. This system will have a resonant frequency of its own which is determined as we learned above, primarily by the amount of load deflection characteristic of the felt pads.

Suppose that a machine in performing its tasks generates internally a rotating out of balance force occurring at 2,000 rpm (33 Hz).

For example, if the machine has a weight of 4800 lbs. and is mounted on (4) LR-1200 Level-Rite Mounts, the LR-1200 has 15 sq. in. of E-1" Red-Line pad material which, under these conditions, will be loaded to 80 psi. From the load deflection characteristics of Type E material we know that the static deflection produced will be about .260" plugging into the relationship of Figure 5 we obtain:

Figure 8

$$F_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{ST}}} = \frac{1}{2\pi}\sqrt{\frac{386.2}{.260}} = 6.13 \text{ Hz}$$

The natural frequency of the system 6.13 Hz is well below the 33 Hz forcing frequency yielding a forcing to natural frequency ratio of 5.5 to 1. The transmissibility from Chart A for this system will be less than .01. Percentage of isolation will be in the high 90's.

THE ISOLATED FOUNDATION

Larger support critical machines requiring isolation (or generating troublesome shocks/vibrations) require the use of an isolated foundation. Here the entire machine and foundation are supported by a deflectable element tuned to provide a natural frequency yielding a desirable forcing to natural frequency ratio as in the previous example. Here the system performance is impacted substantially by the **inertia mass** contributed by the foundation and the high dampening coefficients obtainable through informed system design.

When applied to either sensitive or source machines, the inertia mass substantially reduces the amount of displacement (and hence acceleration - g's) that a given excitation can produce (f = MA). This action, coupled with the basic isolation provided by a properly designed support system as outlined in the previous example, results in a highly effective means of preventing the transmission of troublesome shock and vibrations.

In an actual installation the analysis of the foundation as a suspended mass is substantially more complex than the illustration above as it must be considered as having six degrees of freedom. Also, soil (or other support medium) stiffness must be taken into account as must the energy transmission characteristics of the soil.

The classical equations of motion for a rigid body supported by resilient elements are quite cumbersome, however, if the assumption is made that the body is approximately symmetrical and that the axis of measurement passes through the center of gravity of the system and that the effects of modal coupling on natural frequencies are small, the natural frequency of the system can be expressed:

Figure 9

$$F_z = \frac{1}{2\pi} \sqrt{\frac{\Sigma K_z g}{M}} \simeq 5 \sqrt{\frac{\Sigma K_z}{W}}$$

Where ΣK_z = summation of all spring constants in the direction being considered. This relationship holds for analysis in the horizontal as well as vertical direction and typically produces very acceptable accuracy.

In actual practice it is necessary to incorporate soil stiffness into the analysis. An estimate of the spring constant contributed by supporting soils can be obtained through conventional soil mechanics if basic information concerning the supporting soils is known.

To minimize the impact of soil compliance and assure that maximum isolator efficiency is achieved, it is recommended that soil loadings in a dynamically loaded foundation do not exceed 1/2 the allowable.

This is typically accomplished by adjusting the foundation footprints. For soils whose allowable bearing strength is less than about 6000 psf, a detailed analysis is desirable, as important impact on the overall system's stiffness is highly likely.

Following the design approach outlined above, it is possible through targeting the optimum natural frequency, coefficient of internal dampening and foundation inertia characteristics to achieve excellent system performance. In most cases it is possible to provide substantial reductions in transmitted force and accelerations at or below the system's natural frequency.

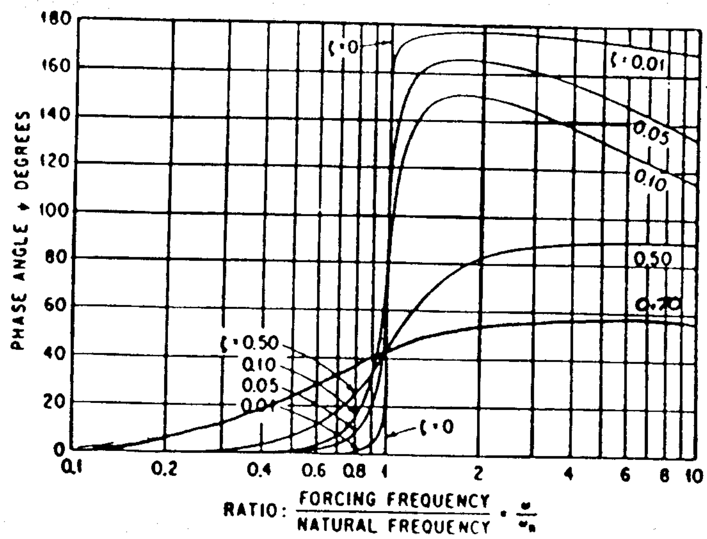


CHART A

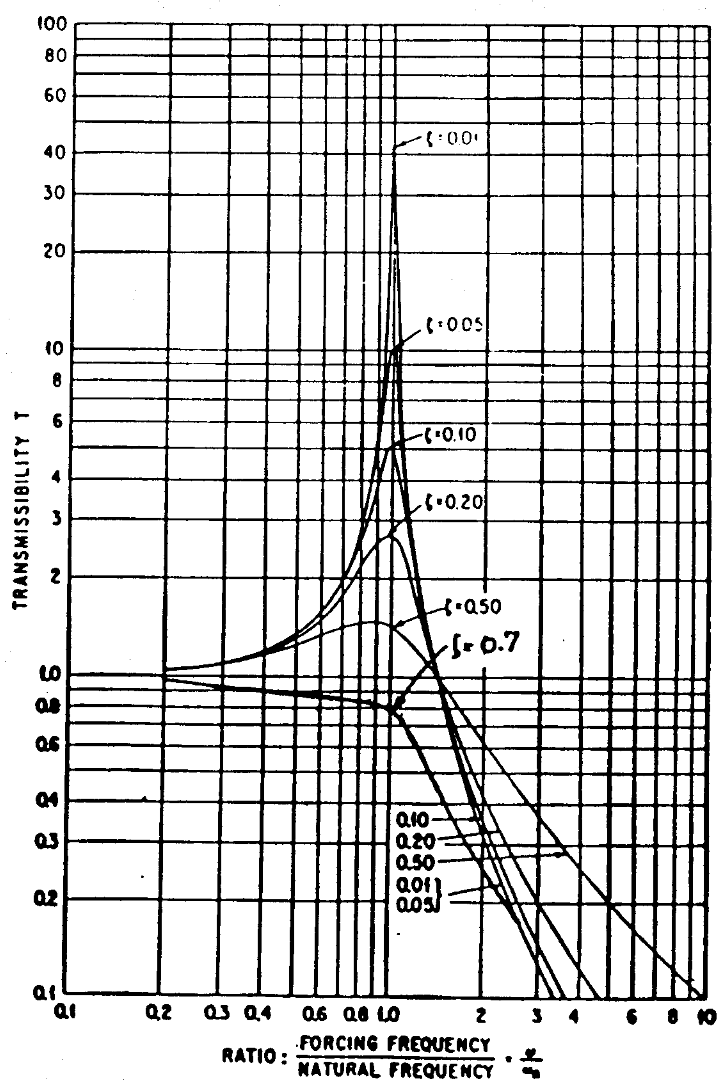
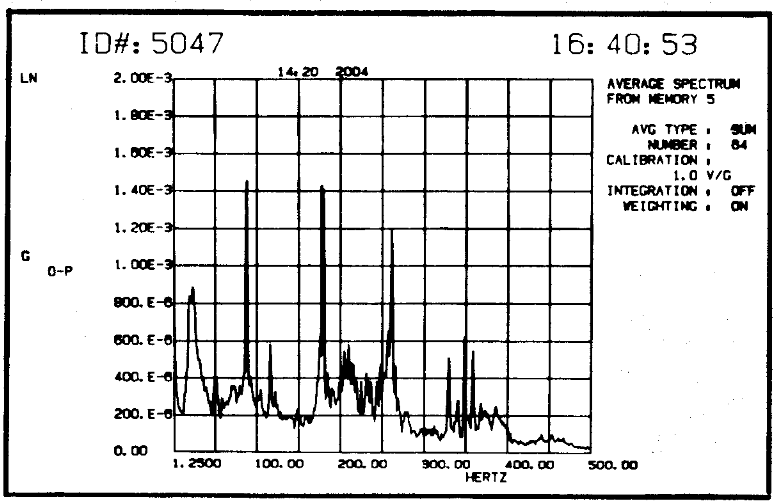


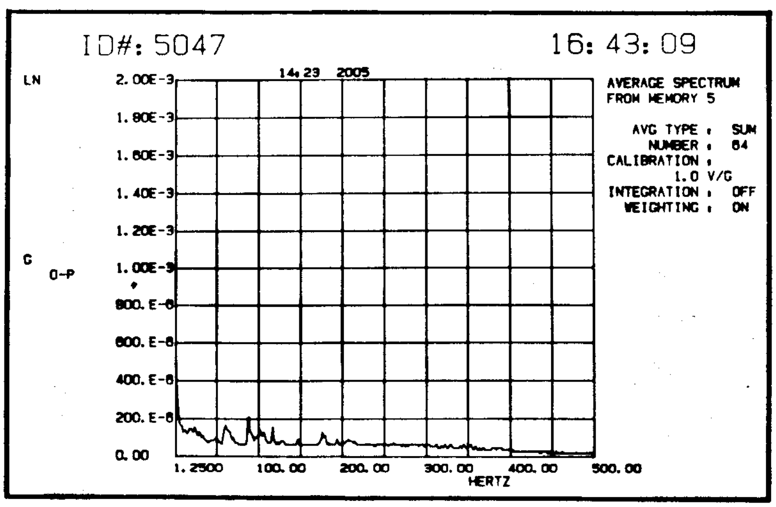
CHART B

INERTIA BLOCK VIBRATION ISOLATION

The charts below show a vibration survey performed by UNISORB on an existing isolated inertia block installation. A grinding machine performing a rough grind operation on a crankshaft was installed on an inertia block isolated as shown on the typical inertia block installation details. During the rough grind the machine-generated vibrations were measured on the foundation. The top chart shows an average of 64 of these measurements.

Vibration measurements were also taken on the concrete floor immediately surrounding the isolated foundation. The lower chart shows an average of 64 of these measurements which were also taken during the rough grind cycle. A comparison of the upper and lower charts is a graphic illustration of the high degree of isolation efficiency provided by UNISORB inertia block isolation materials.





UNISORB® MACHINERY INSTALLATION SYSTEMS

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Vibration transmission from compressor reduced by 75% with isolation system

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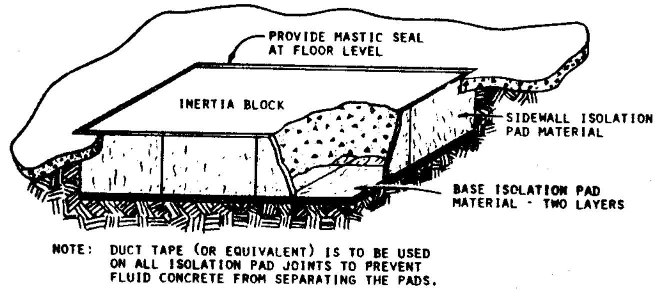

Typical inertia block installation

NEW SOLUTIONS

Problem

Autodie Corporation needed vibration control for the new 200-hp, V-type reciprocating air compressor that was to be installed in a building addition. The compressor, weighing some 7000 lb, is a two-cylinder, two-stage, double-acting, 708-rpm unit, with an operating pressure of 100 psig rated at 1115 cfm and a forcing frequency of 11.8 Hz.

Autodie Corporation makes tools and dies for the automobile industry. The 20-year-old company has 275 employees at its 250,000-sq-ft Grand Rapids, MI facility.

The plant needed some way to isolate the compressor vibration from the new addition building structure so that disturbances to existing precision machine tools would be eliminated.

Solution

Optimum isolation performance depends upon proper isolation materials and installation techniques. The added initial installation costs for vibration isolation are generally considered insignificant when compared to corrective measures at a later date. These results are best achieved through an appropriately sized inertia block effectively isolated from the building structure.

Since the compressor installation was to coincide with the new addition construction, and the stability of the soil was considered satisfactory, the plant selected an isolation pad material that could be used indirect contact with the soil without affecting its properties.

The material consists of a proprietary blend of 100% inorganic synthetic fibers, of various diameters and lengths. The material has no biodegradable components and is environmentally safe. It has a high-tensile-nylon, "sail"-grade fabric cover and a watertight coating that protects the material from moisture migration from the fluid concrete.

The base isolation pad has 900-psi tensile strength and is more dense (.8 lb/sq ft) than the sidewall isolation pad (.7 lb/sq ft) with a tensile strength of 550 psi. Both materials are manufactured in ½"-thick sheets. The plant used two of these ½" sheets under the inertia block and one of them at the sidewall.

The inertia block is reinforced concrete, 2′ thick by 8′ wide and 10′ long. The compressor is rigidly fastened to the inertia block with anchor bolts.

The method the plant used for inertia block vibration isolation is as follows.

First, an excavation the exact size of the finished foundation was prepared. Then two layers of the isolation material were placed in the bottom and one layer on the sides of the excavation. Reinforcing steel was placed and the concrete foundation was poured directly into the cavity.

The installation was completed in the winter of 1982.

Results

On November 10, 1983, vibration analysis tests were conducted to determine the effectiveness of the installation. Readings were taken at several critical locations around the inertia block with an electronic vibration analyzer.

Readings taken at 16 locations (8 on the inertia block and 8 off the foundation) provided an average efficiency of better than 75% transmission reduction.

The plant is satisfied with the results of the vibration control of the installation, and feels that its objectives were met. The compressor and its foundation are decoupled from the surrounding environment.

IB-500 series inertia block vibration isolation material is a product of Unisorb Machinery Installation Systems, Box 1000, Jackson, MI 49204.

Information on reciprocating air compressors can be obtained from Joy Manufacturing Co., Industrial Compressor Group, 301 Grant St., Pittsburgh, PA 15219.

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PRODUCT DATA

UNISORB MACHINERY INSTALLATION SYSTEMS

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BASIC ISOLATED FOUNDATION DESIGN

Check Points

1. For support critical machinery design from allowable deflection--not allowable load. In many applications the foundation becomes part of the machine's structure.

2. For optimum cost/benefit relationship in most cases a mass ratio (foundation to machine) of approximately 1.5 to 1 is desirable.

3. Foundation should not serve as a support for any structure other than the machine to be isolated.

4. Combined center of gravity of the machine and foundation should be below the top of the foundation. The center of gravity of the machine and foundation should pass through the center of the soil pressure diagram within 5% or so of any horizontal dimension.

5. Allowable soil loadings should be verified for the installation with a reduction of 50% being applied when a "source" machine which imparts significant dynamic loadings is being installed. Ideally, a full soils survey should be run at the installation site to permit accurate modeling of the system.

6. When "sensitive" machines are being installed, an ambient site survey (vibration spectrum analysis) should be run under conditions duplicating as closely as possible actual operating conditions.

7. Allowable concrete and steel loads should be in compliance with applicable building codes with appropriate fatigue factors taken into account when high amplitudes are present.

8. Anchor bolt locations should be no closer than 12" to foundation perimeter (or a distance equal to the embedment depth unless special reinforcement for the vertical wall is provided).

9. Contact Unisorb Engineering for no charge computer modeling of the system's performance prior to finalizing design.