

Principles of Foundation Design for Engines and Compressors

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11 BROADWAY NEW YORK, N. Y. 10004

Form 259
PRINTED IN U.S.A.

Principles of Foundation Design for Engines and Compressors

By W. K. NEWCOMB,¹ PAINTED POST, N. Y.

This paper discusses the elastic character of the ground and shows that foundations for reciprocating machines can be treated as spring-supported masses, the machine and foundation representing the mass, and the ground the spring. Such elastic systems have natural periods of vibration; if the frequency of unbalanced inertia or other exciting force is near the natural frequency of the foundation, resonance will occur producing excessive vibration. Examples of this phenomenon are given and tests on foundations having resonance are described. Also, the nature of inertia forces found in reciprocating machines is explained. Examples of good and bad foundation design are shown and a rational method for foundation design is outlined.

INTRODUCTION

THIS paper is an attempt to rationalize foundation design. It is written by a designer and builder of reciprocating machines, one not in the foundation business, and it therefore reflects the observations of a mechanical engineer rather than a civil engineer or expert on soil mechanics.

Foundations for reciprocating machines differ from foundations for buildings or similar structures since dynamic rather than static loads are involved. With a static load, only the bearing capacity of the soil need be considered, and there are various well-known rules to follow. With a dynamic load, however, these rules do not apply, since the frequency of the forces and danger of resonance with attendant excessive vibration must govern foundation design.

ELASTIC CHARACTER OF GROUND

Although ground characteristics and subsoil elasticity have been discussed or mentioned in engineering literature (1, 2, 3, 4, and others)² from time to time, their relationship to the principles of foundation design has not been emphasized adequately nor thoroughly understood. As a result, the elastic character of the ground is not always recognized. The following examples show this characteristic:

An oil-storage tank was observed to settle when filled and rise to its original position when emptied. The test was made with a transit and repeated several times with the same results. There was a definite relation between load and deflection which is typical of elastic materials.

The elastic character of the ground is also shown by a soil-deflection test at another site. A loading platform with 2 sq ft bearing area was loaded gradually. The deflections obtained

are plotted in Fig. 1, curve AB. With 1500 lb per sq ft load, the deflection B, was 0.30 in. This load was maintained for 72 hr and no further deflection took place. Then the load was removed quickly, and the deflection changed to 0.25 in., C. The load was applied quickly, and the deflection went to 0.305 in., D, which is very close to the original deflection, B. The line BC therefore represents the load-deflection characteristics of this ground.

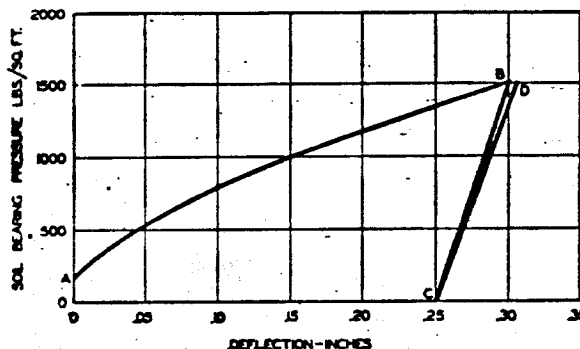


FIG. 1 LOAD-DEFLECTION TEST OF WET SAND AND CLAY SOIL

Because of this elastic nature of the ground the foundation and subsoil form an elastic system consisting of a mass (foundation block and machine) supported by a spring (subsoil). If such an elastic system is excited by periodic forces having a frequency near the natural frequency³ of the elastic system, resonance will occur producing excessive vibration.

FORCES ACTING ON FOUNDATIONS

It is not always possible to obtain perfect balance in reciprocating machines and, as a result, unbalanced periodic forces may exist. These forces are caused by acceleration and deceleration of the piston or other reciprocating parts. Also, centrifugal forces and torque reactions are sometimes factors. Fig. 2 shows the inertia forces in a typical single-cylinder engine or compressor resulting from this acceleration and deceleration. At top dead center (in a vertical machine), the acceleration and resulting inertia force are maximum in the direction away from the crank. At bottom center they are maximum in the opposite direction.

This inertia force F , which acts along the axis of piston or crosshead as shown in Fig. 2(a), can be expressed mathematically as a Fourier series⁴ as follows

$$F = 0.0000284 WRN^2 (\cos \theta + A \cos 2\theta + B \cos 4\theta + C \cos 6\theta + \dots) \dots \dots \dots [1]$$

¹ Mechanical Engineer, Ingersoll-Rand Company. Mem. ASME.
² Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Oil and Gas Power Division and presented at the Oil and Gas Power Conference, Baltimore, Md., June 12-16, 1950, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Manuscript received at ASME Headquarters, March 20, 1950. Paper No. 50-OGP-5.

³ Natural frequency, the frequency at which an elastic system tends to vibrate after being displaced from the equilibrium position and released.

⁴ Reference (5) and other texts on engine dynamics.

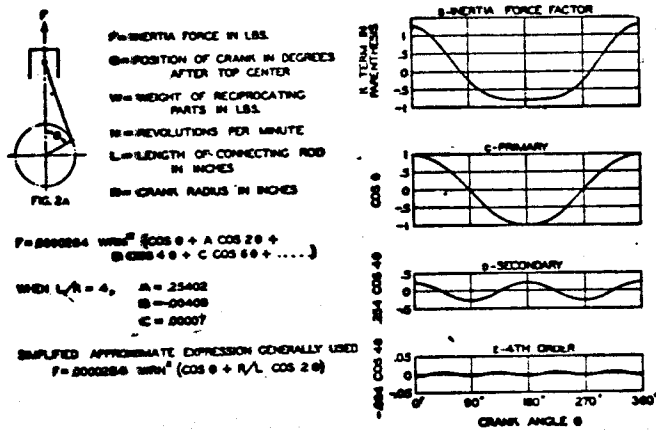


Fig. 2 INERTIA OR ACCELERATION FORCES IN CRANK AND CONNECTING-ROD MECHANISM

where

- F = inertia force, lb
- θ = position of crank in degrees after top or outer dead center
- W = weight of reciprocating parts, lb
- N = speed, rpm
- L = length of connecting rod, in.
- R = crank radius, in.
- A, B, C = constants determined by L/R ratio. When $L/R = 4$, $A = 0.25402$, $B = -0.00409$, $C = 0.00007$

Equation [1] is also written

$$F = 0.0000284 WRN^2 K \dots \dots \dots [2]$$

K is called the inertia force (acceleration) factor and represents the terms in parentheses in Equation [1]. Values of K for different L/R ratios are found in most mechanical-engineering handbooks.

The inertia force F is represented graphically in Fig. 2(b), where K is plotted against crank angle. Note at outer dead center ($\theta = 0$) that K is maximum having a value of 1.250 (when $L/R = 4$). As the crank rotates, K decreases to zero, then becomes negative, reaching a maximum negative value of -0.750 at bottom dead center ($\theta = 180^\circ$). When K is positive, the inertia force F acts away from the crankshaft; when negative it acts toward the crankshaft.

Examination of Equation [1] or [2] shows that the inertia force F varies directly with the reciprocating weight, directly with the stroke (or crank radius) and with the square of the speed.

The first term of the part of the equation in parentheses, that is, "cos θ ," Equation [1], represents the "primary" inertia force which has one complete cycle per revolution of the crankshaft as shown in Fig. 2(c). The second term, " $A \cos 2\theta$," represents the "secondary" inertia force and is much smaller; it has two complete cycles per revolution, Fig. 2(d). The next term is the fourth order with four cycles per revolution, but it is very small and is usually neglected. There are also 6th, 8th, and higher orders, but they are likewise very small and can be neglected. For foundation design we need only consider the primary and secondary forces, and the simplified approximate form of the inertia-force equation is generally used.

Here the inertia force F is expressed as

$$F = 0.0000284 WRN^2 \left(\cos \theta + \frac{R}{L} \cos 2\theta \right) \dots \dots \dots [3]$$

The primary inertia force F' and secondary inertia force F'' become

$$F' = 0.0000284 WRN^2 \cos \theta \dots \dots \dots [4]$$

$$F'' = 0.0000284 WRN^2 \frac{R}{L} \cos 2\theta \dots \dots \dots [5]$$

The primary and secondary forces having different frequencies have different effects on vibration and must be dealt with individually. Since they vary sinusoidally, the maximum values are used for foundation and vibration calculations. The maxima occur when $\cos \theta = 1$ and $\cos 2\theta = 1$, and the primary and secondary forces then become

$$\text{Primary, } F'_{\max} = 0.0000284 WRN^2 \dots \dots \dots [6]$$

$$\text{Secondary, } F''_{\max} = \frac{R}{L} F'_{\max} \dots \dots \dots [7]$$

In single-crank machines both primary and secondary inertia forces are unbalanced. With two cranks at 180 deg the primary force of one crank is opposed to the primary force of the other crank and completely balances it if the reciprocating weights of the two cylinders are the same. The resultant primary force will then be zero. However, the secondary forces act in the same direction and add. With two cranks at 90 deg, the secondaries are balanced and primaries partly balanced. With 3 or more equally spaced cranks, both primary and secondary forces are balanced. However, while the forces may be balanced, couples can be produced by the forces acting along the axes of the dif-

TABLE 1 UNBALANCED INERTIA FORCES AND COUPLES FOR DIFFERENT CRANK ARRANGEMENTS

CRANK ARRANGEMENTS	FORCES		COUPLES	
	PRIMARY	SECONDARY	PRIMARY	SECONDARY
SINGLE CRANK 	F' WITHOUT COUNTERWEIGHTS $0.5F'$ WITH COUNTERWEIGHTS	F''	NONE	NONE
TWO CRANKS AT 180° IN LINE CYLINDERS 	ZERO	$2F''$	$F'd$ WITHOUT COUNTERWEIGHTS $F'd$ WITH COUNTERWEIGHTS	NONE
OPPOSED CYLINDERS 	ZERO	ZERO	NIL	NIL
TWO CRANKS AT 90° 	$0.41F'$ WITHOUT COUNTERWEIGHTS $0.707F'$ WITH COUNTERWEIGHTS	ZERO	$0.07F'd$ WITHOUT COUNTERWEIGHTS $0.354F'd$ WITH COUNTERWEIGHTS	$F'd$
TWO CYLINDERS ON ONE CRANK CYLINDERS AT 90° 	F' WITHOUT COUNTERWEIGHTS ZERO WITH COUNTERWEIGHTS	$0.41F''$	NIL	NIL
TWO CYLINDERS ON ONE CRANK OPPOSED CYLINDERS 	$2F'$ WITHOUT COUNTERWEIGHTS F' WITH COUNTERWEIGHTS	ZERO	NONE	NIL
THREE CRANKS AT 120° 	ZERO	ZERO	$0.48F'd$ WITHOUT COUNTERWEIGHTS $0.3F'd$ WITH COUNTERWEIGHTS	$0.48F'd$
FOUR CYLINDERS CRANKS AT 180° 	ZERO	$4F''$	ZERO	ZERO
CRANKS AT 90° 	ZERO	ZERO	$1.41F'd$ WITHOUT COUNTERWEIGHTS $0.707F'd$ WITH COUNTERWEIGHTS	$4.0F'd$
SIX CYLINDERS 	ZERO	ZERO	ZERO	ZERO

F' = PRIMARY INERTIA FORCE IN LBS.
 $F' = 0.0000284 RN^2 W$
 F'' = SECONDARY INERTIA FORCE IN LBS.
 $F'' = \frac{R}{L} F'$
 R = CRANK RADIUS, INCHES
 N = R.P.M.
 W = RECIPROCATING WEIGHT OF ONE CYLINDER, LBS
 L = LENGTH OF CONNECTING ROD, INCHES
 d = CYLINDER CENTER DISTANCE

ferent cylinders. Table 1 shows unbalanced forces and couples obtained with some common crank arrangements.

RESONANCE IN FOUNDATIONS

Since the foundation is an elastic system, these periodic forces (and to a lesser extent the couples) tend to induce vibration. If the frequency of the exciting force is near the natural frequency of the foundation, resonance will occur and may cause excessive vibration.

Fig. 3 shows a gas-engine-driven compressor foundation having resonance within the operating speed range. While adequate concrete yardage was used (four times that generally necessary for a machine of this type), it was not placed effectively. The

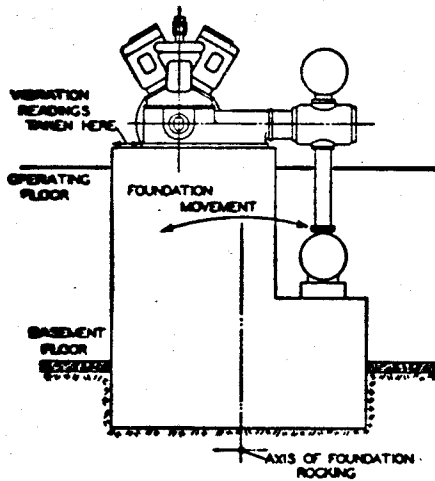


FIG. 3 FOUNDATION HAVING RESONANCE IN OPERATING SPEED RANGE

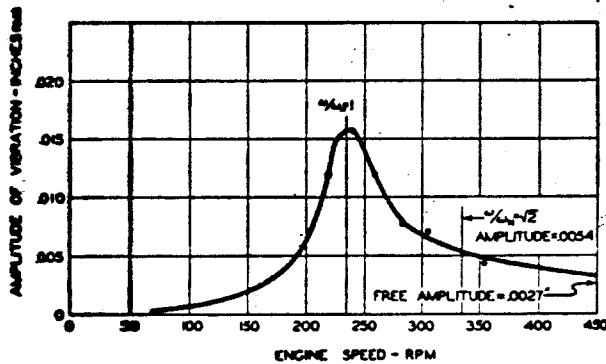


FIG. 4 VIBRATION TEST SHOWING RESONANCE IN A FOUNDATION

ground was soft, and the result was excessive vibration. Vibration measurements showed that the foundation rocked back and forth about an axis slightly below the base as marked in Fig. 3. This movement was caused by horizontal secondary forces in the compressor.

The horizontal movement at top of the foundation is plotted in Fig. 4. Note that at 354 rpm the amplitude of vibration is 0.0045 in. As the speed decreases the amplitude increases, reaching a maximum of 0.016 in. at 236 rpm, although the inertia force (which varies as the square of the speed) is 56 per cent less than at 354 rpm. Below 236 rpm the amplitude drops off sharply. In this particular machine the unbalanced force was 10,800 lb (secondary) at 354 rpm, and 4800 lb at 236 rpm.

The vibration peaking in this manner is typical of an elastic system. As there were two complete vibrations per revolution

due to unbalanced secondary forces, the natural frequency is 472, the vibrations per minute occurring at 236 rpm, the resonant speed.

An engine or compressor foundation can be represented graphically as a spring-supported mass with a dashpot to provide damping as shown in Fig. 5. When the mass, spring characteristics, and damping are known, also the magnitude and frequency of the exciting force, the amplitude of the resulting vibration can be determined.

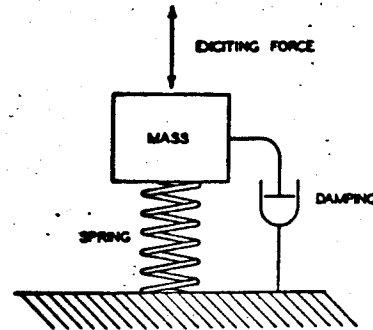


FIG. 5 FOUNDATION REPRESENTED AS A SPRING-SUPPORTED MASS

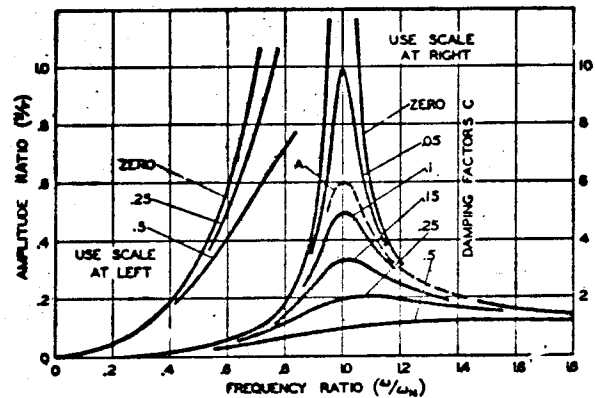


FIG. 6 VIBRATION-RESONANCE CURVES

Fig. 6 shows typical resonance curves for the elastic system represented in Fig. 5. Here "amplitude ratio" is plotted against "frequency ratio" for different damping factors.⁸ Note that

$$\text{Amplitude ratio, } \frac{y}{r} = \frac{\omega^2/\omega_n^2}{\sqrt{(1 - \omega^2/\omega_n^2)^2 + (2c\omega/\omega_n)^2}} \dots\dots\dots [8]$$

when

- y = actual amplitude
- r = free amplitude, the amplitude with which the foundation mass would vibrate if it were not restrained
- c = damping factor, viscous damping assumed
- ω = frequency of exciting force
- ω_n = natural frequency of foundation

$$\text{Free amplitude, } r = 35,200 F/W\omega_n^2, \text{ in.} \dots\dots\dots [9]$$

⁸ Damping is the reduction in amplitude of vibration due to friction or viscosity, etc. Viscous damping is assumed where the damping is proportional to the square of the velocity (9). When the damping is sufficient to just prevent vibration, c = 1. When c = 0, the amplitude of vibration at resonance becomes infinite.

⁹ Reference (7) and other texts on vibration.

where

- F = maximum value of primary or secondary inertia force, whichever is being considered, lb
- W = combined weight of foundation and machine, lb
- ω = frequency of inertia force F , in cycles per min (cpm)

An examination of Fig. 6 shows the amplitude ratio is zero, when the frequency ratio is zero, which means there is no vibration when the machine is standing still. Then, as the speed of the machine and frequency and magnitude of the exciting force are increased, the amplitude increases, reaching a maximum at or near the frequency ratio of 1 (for the smaller damping factors such as $c = 0.25$ or less). At this speed the periodic exciting force is in resonance with the natural frequency of the foundation. At higher speeds the amplitude falls off and approaches the free amplitude. Let us see how these resonance curves can be applied to different types of foundations.

There are two general types of foundations; (a) the resiliently mounted foundation supported on springs, rubber, cork or felt, etc., and, (b) the conventional concrete foundation poured directly on the ground, with or without piling. While this paper mainly deals with the second type, it may be of interest to point out some differences between them so as to emphasize the characteristics of the latter. To be effective, the resilient foundation must have a natural frequency well below the frequency of any exciting force so that the frequency ratio is 2 or 3.⁷ On the other hand, it is desirable that the conventional foundation have a frequency ratio of 0.5 or less to avoid excessive vibration.

Damping factors of soil c , vary widely, an average value (1) being 0.25. However, the foundation in Fig. 3 had a damping factor of 0.082, which is unusually low and was a result of the rocking action displacing little of the ground. The vibration measurements of this foundation are plotted in Fig. 6, curve A, showing that the actual performance agrees with vibration theory. Fortunately, it is not necessary to know the damping factor of the soil because in a good foundation the frequency ratio is 0.5 or less. Note that the damping has little effect on the amplitude in this part of the resonance curve, and if neglected the error will be small and on the safe side. Thus, to predict the performance of a foundation it is necessary to know only the "free amplitude," "natural frequency," and the "magnitude" and "frequency" of the "exciting forces."

FACTORS AFFECTING NATURAL FREQUENCY OF A FOUNDATION

In theory a foundation can have six degrees of freedom. That is, it may vibrate in six different ways as shown in Fig. 7, or a combination of them. However, in the actual foundation the movement is generally either horizontal or vertical, depending on which force predominates or is near resonance. The horizontal

Fig. 7. Vibrations (7, 8).

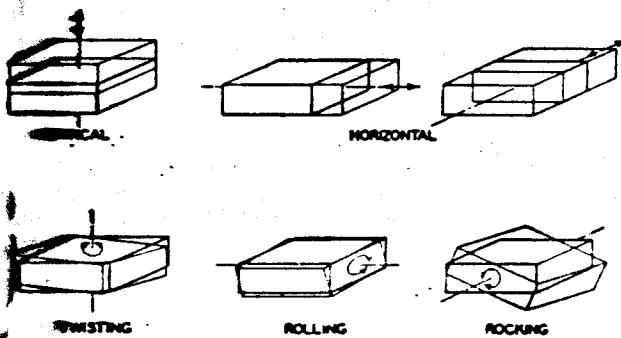


Fig. 7. VIBRATION OF FOUNDATIONS, SIX DEGREES OF FREEDOM

natural frequency is usually the same as the vertical natural frequency. The natural frequency in rocking is influenced by the dimensions of the foundation. For shallow foundations the rocking natural frequency will be very nearly the same as the vertical natural frequency. For high foundations (also deep foundations) it will be much lower. The approximate natural frequency of a foundation can be determined from the static deflection as described later in Equation [10] or expressed in terms of soil-bearing pressure as shown in Fig. 8.⁸ Here the

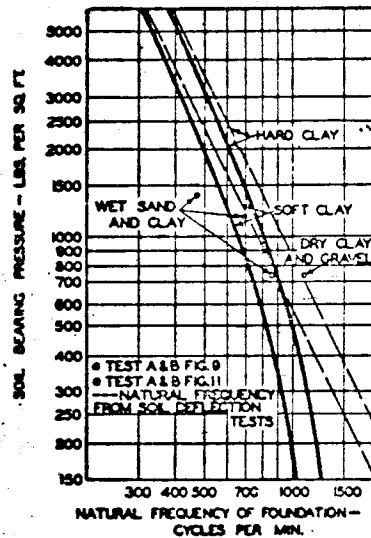


Fig. 8. NATURAL FREQUENCY OF FOUNDATION ON TWO TYPES OF SOIL

natural frequencies for two different soils are given. Note that the softer ground has the lower natural frequency. The graph also shows how the natural frequency is affected by soil loading; the lower the soil bearing pressure, the higher the natural frequency. This fact provides a means for controlling the natural frequency. If the natural frequency of a proposed foundation is too low, it can be raised, within limits, by reducing the soil bearing pressure. This improvement can be accomplished by spreading the foundation or placing the foundation block on a mat.

Fig. 9 is an example of raising the natural frequency of a foundation by extending the base. This foundation is the same one shown in Fig. 3, but a reinforced mat, 2 ft thick, was added to tie to another foundation on one side. This change raised the resonant peak ($\omega/\omega_n = 1$) from 236 rpm, curve A, to 346 rpm, and substantially reduced the amplitude as shown in curve B. The new natural frequency is 692 vibrations per min and is a big improvement over the original foundation.⁹ Curve A (same as Fig. 4), shows amplitude of the original foundation for comparison.

⁸ Fig. 8, while empirical, is based on published data on soils. The upper ends of the curves, where the soil bearing pressure is high, approach the natural frequencies calculated from static deflection (1). The lower ends of the curves where the soil bearing pressure is low, approach the natural frequency of the ground determined by vibrator tests, such as (2). Thus it takes into account the fact that the ground acts like a spring having appreciable mass. Reference (4) gives additional information on natural frequency of foundations.

⁹ The shape of the resonance curves in this test indicates substantially linear soil-deflection characteristics although other investigators (3) find nonlinearity. This difference may be the result of size or shape of foundation or magnitude of deflection of the soil. The broken line in curve B is superimposed vibration about another axis.

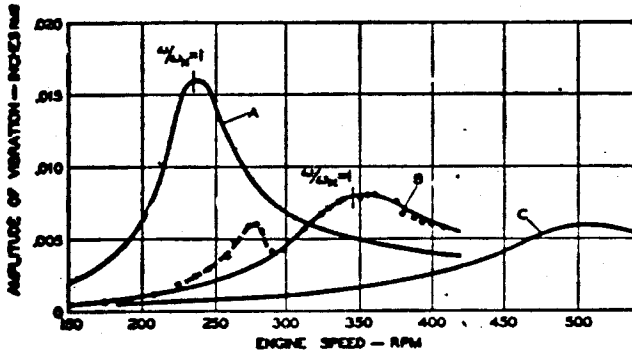


FIG. 9 RAISING NATURAL FREQUENCY BY EXTENDING FOUNDATION BASE

The revised foundation was satisfactory for all speeds up to 300 rpm. Further improvement could be obtained by extending the base on the other side. Curve C, Fig. 9, shows results expected if the foundation had been placed originally on an adequate mat, so as to provide a lower soil bearing pressure and obtain a higher natural frequency.

Fig. 10 shows an example of resonance in a compressor foundation.

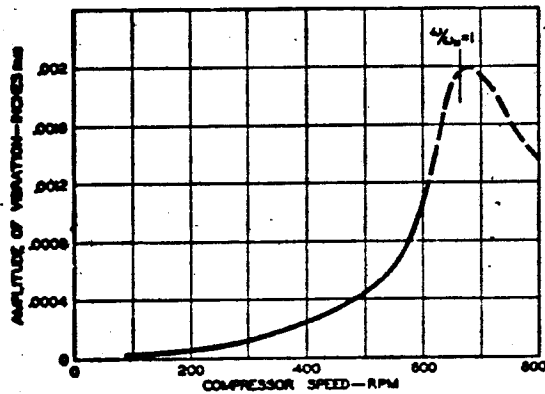


FIG. 10 RESONANCE IN COMPRESSOR FOUNDATION

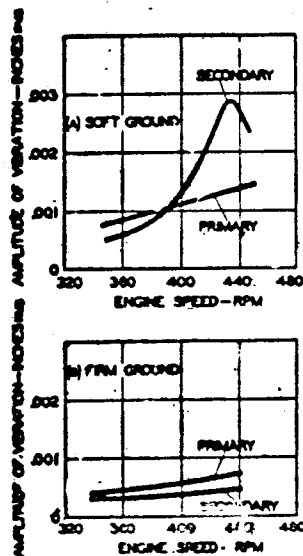


FIG. 11 RESONANCE IN ENGINE FOUNDATION

The effect of soft and firm ground under identical foundations is shown in Fig. 11. Fig. 11(a) shows the results of a vibration test where a foundation intended for firm ground was placed on soft ground. Here the natural frequency was too low, producing resonance and excessive vibration at 434 rpm. In Fig. 11(b), the engine and foundation were exact duplicates, but the ground was firm. A much higher natural frequency (beyond the range of the test) was obtained and vibration was nil.

The desirability of low soil loadings for soft ground, such as wet sand and clay, cannot be overemphasized. A common mistake in the design of engine and compressor foundation is to follow the soil bearing pressures allowed by building codes such as Table 2. These soil bearing pressures are much too high for dynamic loads. Soft clay or wet sand, for instance, is considered suitable for 2000 lb per sq ft under building footings, but an engine or compressor foundation on this soil and with this bearing load will have a natural frequency of 490 vibrations per min (from Fig. 8), which is suitable only for a low-speed machine.

TABLE 2 EXAMPLES OF ALLOWABLE SOIL BEARING PRESSURES FOR STATIC LOADS

	Lb per sq ft
Quicksand or alluvial soil.....	1000
Soft clay, sand, loam, silt.....	2000
Firm clay, sand and clay.....	4000
Hard clay.....	6000
Compact sand and gravel.....	8000
Shale and hard pan.....	18000
Rock.....	20000 and higher

Note: For dynamic loads only a fraction of these soil bearing pressures can be used depending on frequency of forces present.

The fact that high or deep foundations have a lower natural frequency in rocking than shallow foundations has already been mentioned. A comparison of typical shallow and high foundations shows that the natural frequency of the latter may be reduced to one half, unless special precautions are taken. When necessary to install engines or compressors on high foundations a mat should be used to lower the soil loading and raise the natural frequency. The block on which the machine rests can be relatively small, the only requirement being that it is sufficiently strong and rigid to provide adequate support and maintain proper alignment of the machine and transmit the vibration forces to the mat.

PILING

When poor soil is encountered, piling is often desirable. It is beyond the scope of this paper to discuss the detailed use of piles as this subject belongs to the foundation specialist. However, piles properly used will raise the natural frequency of the foundation, especially the vertical natural frequency if driven to firm ground. When substantial horizontal inertia forces are encountered and piling is necessary, the use of batter piles is important to keep the natural frequency high. A foundation supported on piles should always be checked for natural frequency on the assumption that the earth may shrink away leaving the foundation supported on columns. This can be done by determining the "static deflection" caused by a horizontal force equal to the total weight of the foundation and machine. There is a convenient relation between the static deflection and the natural frequency as follows

$$\omega_n = 188 \sqrt{\frac{1}{\delta}} \dots \dots \dots [10]$$

where

ω_n = natural frequency, vibrations per min
 δ = static deflection, in.

Fig 12 shows natural frequency plotted against static deflection.

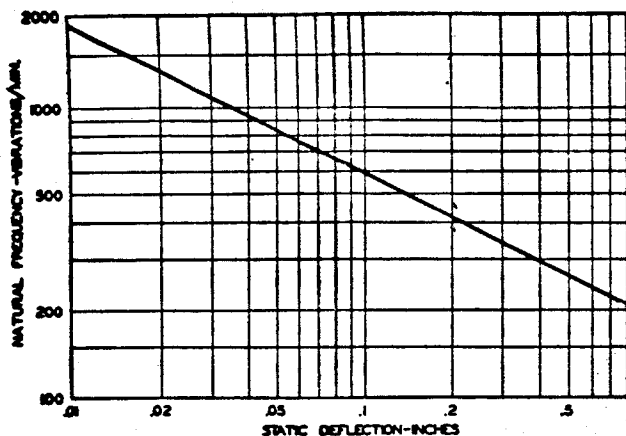


FIG. 12 RELATION BETWEEN STATIC DEFLECTION AND NATURAL FREQUENCY

DESIGNING FOUNDATIONS

Manufacturers' foundation drawings show foundations suitable for firm soils such as well-cemented sand and gravel or hard clay. When softer ground is encountered, additional precautions must be taken. Many foundation problems can be solved by placing the foundation block on a mat, the size of which is determined by the magnitude and frequency of the unbalanced forces and the character of the ground. In other cases piling can be used effectively. It is good practice to refer the design of all special foundations to a foundation expert, one familiar with soil dynamics.

In designing the foundation for an engine or compressor, or any machine having periodic forces, the procedure to follow should be:

- 1 Determine the magnitude and frequency of the unbalanced forces. This information is generally supplied by the manufacturer of the machine. The frequency of the unbalanced forces establishes what the natural frequency of the foundation must be to avoid resonance and obtain a vibrationless installation. The natural frequency should be at least twice the frequency of any substantial unbalanced force. Expressing it another way, the frequency ratio should be less than 0.5.

- 2 Determine the character of the soil by borings, deflection tests, and, if possible, dynamic tests (2, 3, 6). The maximum allowable bearing pressure required to obtain the natural frequency found in item 1 can be developed from these data or an approximate value can be read directly from Fig. 8.

- 3 Pick off the amplitude ratio from Fig. 6.

- 4 Assuming some allowable amplitude of vibration such as 0.002 in., determine the free amplitude.

- 5 Knowing the magnitude and frequency of the unbalanced force, item 1, and the free amplitude, item 4, the total mass of the machine and foundation can be found. But the foundation must be proportioned so as not to exceed the maximum allowable soil bearing pressure determined in item 2.

THE IDEAL FOUNDATION

Fig. 13 shows examples of good and bad foundation design. Avoid high or deep foundations. If the latter type must be used be sure to provide a generous mat. The ideal foundation is shallow (but sufficiently rigid to maintain alignment) and is spread out to obtain a low soil bearing pressure and place a large

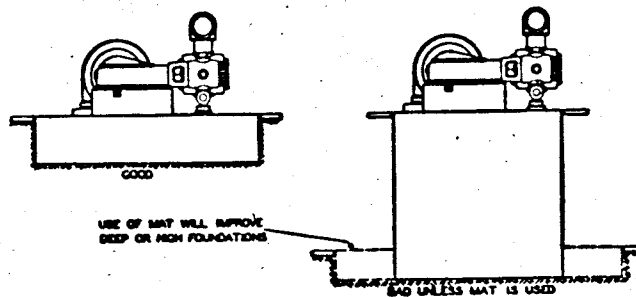


FIG. 13 EXAMPLES OF GOOD AND BAD FOUNDATIONS FOR A HORIZONTAL COMPRESSOR

surface in contact with the ground. This construction insures a high natural frequency which is necessary for vibrationless operation.

ACKNOWLEDGMENTS

The author wishes to express his appreciation to members of the Ingersoll-Rand Company staff for assistance with the preparation of this paper. Also, to Mr. F. E. Rasmers for data on soil deflection, and to Mr. W. P. Kinneman for suggestions on use of piling.

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Appendix

INSTRUMENTS USED TO MEASURE VIBRATION

The vibration tests described in this paper were made with the Type 761A vibration meter and Type 762B vibration analyzer made by the General Radio Company, Cambridge, Mass. These instruments are inertia-operated crystal type and measure the root-mean-square (rms) amplitude which is plotted in the different illustrations. The maximum amplitude from the mean position is 1.414 times the rms amplitude, while the total over-all displacement (sometimes called double amplitude) is 2.828 times the rms amplitude.

The test, shown in Fig. 10, was made with the foregoing vibration-measuring instruments and recorded with a Brush Development Company oscillograph.