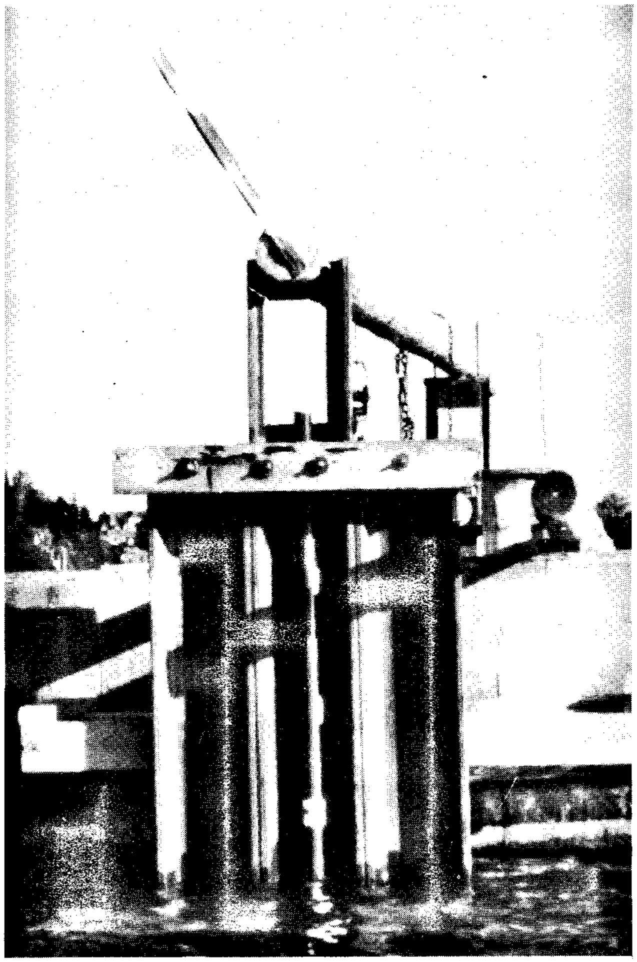
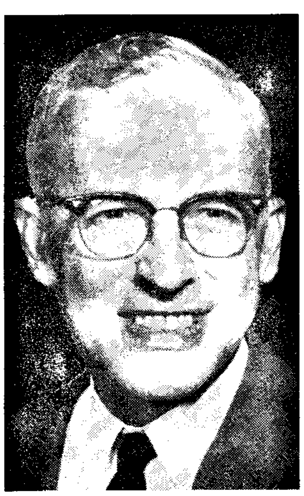


Sheet pile wall used in lateral pile load test in peat. Slope Indicator casing is attached along the full length of the center pile. An inclinometer is lowered down this casing to measure the change in inclination of the piles.



Stanley D. Wilson

David E. Hilts

How To Determine Lateral Load Capacity Of Piles

By S. D. Wilson and D. E. Hilts

Stanley D. Wilson, P.E., Vice President
David E. Hilts, P.E., Senior Engineer
Shannon & Wilson, Inc., Soil Mechanics and Foundation Engineers
Seattle, Washington

Mr. Wilson has served as consultant in many important hydro-electric projects around the world and is currently active in rock-fill dam design in Mexico. He is an authority on instrumentation for field measurements and is president of the Slope Indicator Company which develops new types of instrumentation.

Mr. Wilson received his S.M. degree from Harvard and has been associated with Shannon and Wilson since the firm's inception in 1954. Winner of an ASCE Research Award in 1961, he is a fellow in the ASCE and former chairman of the Soil Mechanics and Foundation Division. He is also a member of ASTM, U.S. Commission on Large Dams (USCOLD), American Institute of Consulting Engineers, Consulting Engineers Council (CEC), the Boston Society of Civil Engineers (BSCE), and the Harvard Society of Engineers and Scientists. He was recently elected to membership in the National Academy of Engineering.

Mr. Hilts became associated with the firm in 1963 and is a registered professional engineer in the State of Washington. He received both his BSc in Civil Engineering and MSc in Soil Mechanics from Queen's University in Kingston, Ontario, Canada. He is an associate member of ASCE. During his association with Shannon and Wilson, Mr. Hilts has been project engineer for many large projects including post-earthquake studies in Valdez and Seward, Alaska and stabilization studies for the Turnagain landslide in Anchorage, Alaska.

PILES are often required to resist lateral forces resulting from wind, waves, earth pressures, earthquakes, or the lateral thrust of an inclined and/or horizontal structural member. It is necessary, therefore, to be able to predict with a reasonable degree of assurance, just how a pile will react when subjected to a horizontal thrust, and in particular, to ensure that both the deflection of the top of the pile and the stresses within the pile will not exceed prescribed limits.

Solution For Laterally Loaded Piles

Procedures for evaluating the behavior of a pile when subjected to lateral loads are not firmly established. Many useful approaches, however, have appeared in the literature within the past few years. Most design methods assume an elastic representation of the pile and surrounding soil and are derived from various solutions of the governing equation (1),*

$$\frac{d^4y}{dx^4} + \frac{k}{EI} \cdot y = 0 \qquad \ldots 1$$

where y is the lateral deflection of the pile at depth x below the ground surface (Fig. 1), EI is the flexural

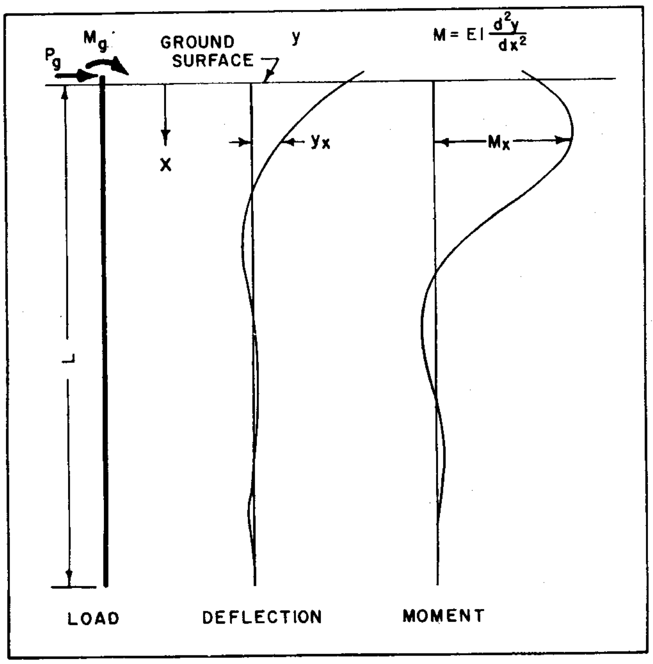


FIG. 1 TYPICAL DEFLECTION AND MOMENT CURVES FOR LATERALLY LOADED PILE

stiffness of the pile and k is a measure of the stiffness of the soil surrounding the pile. The coefficient, k, is termed the "modulus of subgrade reaction" or the "subgrade modulus." By definition, k represents the ratio of the soil reaction at a point on the surface of the pile to the corresponding deflection at the same point. Algebraically,

$$k = \frac{-w}{y} \qquad \ldots 2$$

where w is the resultant soil reaction across the width of the loaded surface. The units of k are force/length². The minus sign in equation 2 signifies that the directions of w and y are opposite.

The solution of equation 1 depends on the assumptions made regarding the variation of the subgrade modulus with depth. Solutions are available (2) for any fixed variation of k with depth but a computer generally is required. For most practical problems it is sufficient to assume that k is either 1.) constant for all depths, or 2.) varies lineally with depth. In conjunction with the latter it is convenient to define this variation by,

$$k = n_h \cdot x \qquad \ldots 3$$

where n_h is termed the "constant of horizontal subgrade reaction" and has the units, force/length³.

Reese and Matlock (3) have presented a nondimensional solution for the special case defined by equation 3. The equations defining the various functions (deflection, slope, moment, etc.) are summarized in Table 1. Typical charts for determining the moment and deflection coefficients are presented in Fig. 2; a

TERM	EQUATION	
	$k = n_h \cdot x$	k = constant
a. Depth coefficient	$z = \frac{x}{T}$	$z = \frac{x}{R}$
b. Max depth coefficient	$Z_{max} = \frac{L}{T}$	$Z_{max} = \frac{L}{R}$
c. Relative stiffness factor	$T = \sqrt[5]{\frac{EI}{n_h}}$	$R = \sqrt[4]{\frac{EI}{k}}$
d. Deflection, y	$y = A_y \frac{P_g T^3}{EI} + B_y \frac{M_g T^2}{EI}$	$y = A'_y \frac{P_g R^3}{EI} + B'_y \frac{M_g R^2}{EI}$
e. Slope, θ	$\theta = A_s \frac{P_g T^2}{EI} + B_s \frac{M_g T}{EI}$	——
f. Moment, M	$M = A_M P_g T + B_M M_g$	$M = A'_M P_g R + B'_M M_g$
g. Shear, V	$V = A_V P_g + \frac{B_V M_g}{T}$	——
h. Soil reaction, w	$w = A_w \frac{P_g}{T} + B_w \frac{M_g}{T^2}$	——

Note: For a complete set of influence charts for the A,B and A',B' coefficients refer to references (3) and (4) respectively

TABLE I SOLUTIONS OF EQUATION 1 FOR k = constant AND $k = n_h \cdot x$

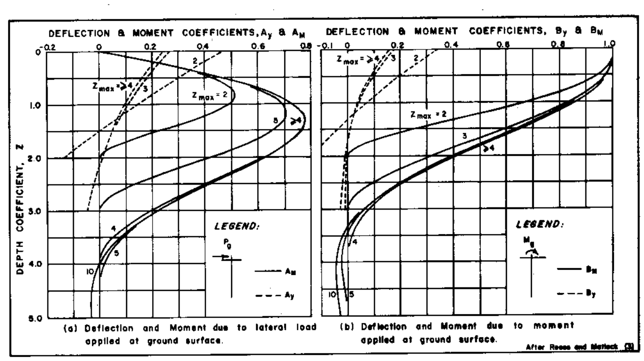


FIG. 2 INFLUENCE CURVES FOR DEFLECTION AND MOMENT OF FREE-HEAD PILE WHEN $k = n_h \cdot x$

complete set of charts is given in reference (3). A similar solution in which k is constant has been prepared by Davisson and Gill (4). The equations from this solution are similar to those for the previous case and differ only in the definition of the relative stiffness factor (Table 1).

The curves in Fig. 2 reveal that for values of Z_{max} equal to or less than 2, the pile behaves as if it were infinitely rigid, and the deflection can be described almost as pure rotation about some axis on the embedded portion of the pile. In such cases a pile is referred to as a pole. This distinction is made because the design procedure for poles generally is based on limiting soils resistance as compared to the elastic analysis represented by equation 1. A discussion of pole design is beyond the scope of this article. How-

*Numbers in parentheses refer to the reference list at the end of this article.

**When determining the deflection coefficients Ay and By, multiply the values read from the graph by 10. No correction is necessary for the Moment coefficients.

ever, the interested reader is directed to reference (5) for a complete discussion on soil-pole behavior.

Determination of the Subgrade Modulus

A comprehensive treatise on the subject of subgrade reaction was presented by Terzaghi (6) in 1955 in which he discussed the various factors affecting the variation and magnitude of the modulus of subgrade reaction. Terzaghi suggested that for preloaded cohesive soils k may be assumed constant. However, Davisson and Gill (4) show that in some cases this assumption may result in an understatement of the surface deflection and maximum moment by a factor of 2. Fortunately, the majority of problems do not fall into this category. Terzaghi further recommended that for sand, k varies lineally with depth beginning with zero at the ground surface. The results of lateral pile load tests reveal that this assumption is equally applicable to normally loaded cohesive soils. Recommended values for the constant of subgrade reaction, n_h, for granular soils and for normally loaded cohesive soils are given in Tables 2 and 3, respectively.

SOURCE	LOOSE		MEDIUM		DENSE	
	DRY	SUBMERGED	DRY	SUBMERGED	DRY	SUBMERGED
Terzaghi (6)	8.1	4.7	24	17	65	40
Rowe (9)	2.9	—	—	—	92	—
Davisson & Parkash (5)	1.5[a]	—	—	—	100	—

a. Cyclic loading

TABLE 2 TYPICAL VALUES OF n_h (pci) FOR SANDS

SOURCE	n_h (pci)	SOIL DESCRIPTION
Davisson & Robinson (7)	67 S_u^a	Normally loaded cohesive soil
Davisson & Prakash (5)	1.0^b- 2.0	Soft clay
Peck & Davisson	0.4 - 1.0	Soft organic silt
Reese & Matlock (3)	0.6 - 12.7	Soft clay
Shannon & Wilson[c]	0.1 - 0.4	Submerged, fibrous peat

a S_u = undrained shear strength

b repetative loading

c from job file

TABLE 3 TYPICAL VALUES OF n_h (pci) FOR NORMALLY LOADED COEHESIVE SOILS

The most reliable procedure for determining the value of n_h (or k) is by performing lateral pile load tests. Often, these tests can be performed in conjunction with conventional pile load tests. In the simplest form the test involves jacking horizontally on the top of the pile and measuring the resulting deflection or slope at the top of the pile. Measurements are made at various loads and often the load may be cycled several times. The results are then equated to an appropriate pile theory and a value of n_h (or k) is determined. It is necessary, of course, to presume some fixed variation of k with depth. In more elaborate tests the deflected shape and/or stresses within the pile are measured at various depths and a more realistic variation k with depth is obtained.

The writers had the opportunity to conduct such a test to determine the coefficient of subgrade reaction of a thick deposit of soft peat. The tests were performed by jacking between two rows of sheet piles embedded in the peat. The sheet piles were selected because of their flexibility and had no bearing on the design piles. The change in inclination of the piles due to various loads was determined at one-foot increments along the full length of each pile. The field results were equated to the Reese and Matlock solution (Table 1) and as shown in Fig. 3 compare extremely well with the theory.

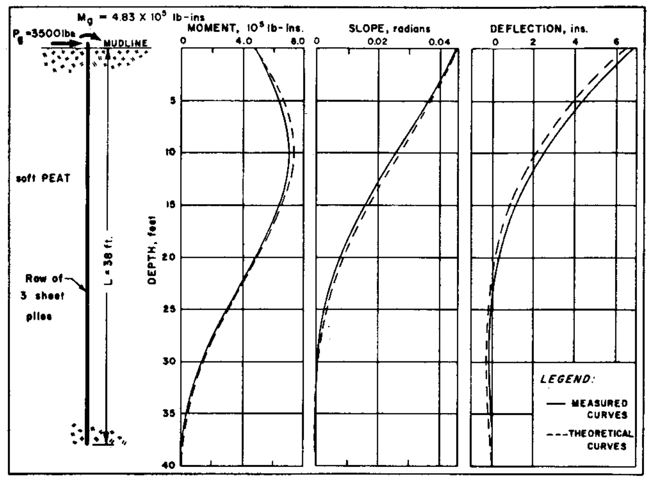


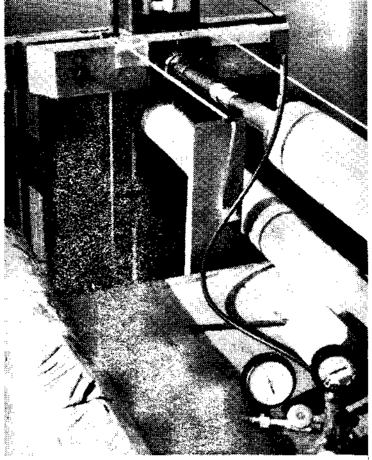
FIG. 3 TYPICAL RESULTS FROM LATERAL PILE LOAD TEST IN SOFT PEAT

Cyclic loading and pile spacing must be considered in the selection of an appropriate k. Cyclic loading may produce deflections at the ground surface equal to twice those which occur from the first application of the load. Further, pile spacing equal to or less than about three pile diameters may reduce the value of k as low as 25 percent of the value measured for a single pile. However, pile spacing in excess of eight pile diameters appears to have no effect on the value of k (8).

The selection of the appropriate soil properties is probably the most important single factor affecting the accuracy of the theoretical solution. The theory by definition assumes that the soil behaves as an ideal elastic material. This is, at best, only a crude approximation. This problem is not unique to laterally loaded piles. Nevertheless, the selection of the soil properties to be used in any lateral load analysis, even when the results of a pile load test are available, should be left to the experienced soils engineer.

Design Procedures

The use of the preceding solution for laterally loaded piles is illustrated by the following example involving the design of small dock facilities and floating walkways across a shallow waterway. The design engineer wished to anchor the floating structures with vertical timber piles embedded in the soft peat which underlay the site to depths in excess of 40 feet. The conditions of the



A twenty-ton jack provides the lateral load. A similar row of sheet piles on the other side of the working platform provides the reaction. The cable is attached to the inclinometer which has been lowered into the Slope Indicator casing.

problem are shown in Fig. 4. To minimize sidesway it was deemed necessary to limit the deflection at the mudline to 1 or 2 inches. To assist the structural engineer we prepared the curves in Fig. 5 which relate the lateral load capacity and maximum bending moment to various depths of penetration for class B treated timber piles assuming an allowable deflection of 1 inch at the mudline. These curves were determined in the following manner. The computations shown are for 20 feet of penetration only.

The properties of the timber pile were assumed to be: E = 1.5 x 10^6 psi; tip diameter, 8 ins.; and taper, 1 in. per 10 ft. of length. The maximum moment generally occurs in the upper 5 to 10 feet, hence the average moment of inertia was computed from the diameter of the pile at a depth of 6 feet and found to be 383 in.^{4} Therefore EI = (1.5 x 10^6) (383) = 5.74 x 10^8 lb.-in.^{2}

The results of a lateral pile load test performed in the area as part of another project revealed that n_h for the peat was about 0.4 lbs/in.^{3} for small strains at the mudline. Since the piles most likely would be subjected to cyclic loading, a value of n_h = 0.2 lbs/in.^{3} was selected for design.

Therefore,

$$T = \sqrt[5]{\frac{EI}{n_h}} = \sqrt[5]{\frac{5.74 \times 10^8}{0.2}} = 77.8 \text{ ins.}$$

From Table 1, equation d,

$$Y = A_y \cdot \frac{PT^3}{EI} + B_y \cdot \frac{MT^2}{EI} = 1 \text{ in.}$$

From Fig. 2 at z = 0 and Z_{max} = (20) (12)/77.8 = 3.08 (use Z_{max} = 3) the coefficient A_y and B_y are 2.70 and 1.75, respectively. Hence,

$$\frac{(2.70)\,(P)\,(77.8)^3}{5.74 \times 10^8} + \frac{(1.75)\,(96P)\,(77.8)^2}{5.74 \times 10^8} = 1 \text{ in.}$$

from which P = 250 lbs.

To determine the magnitude and location of the maximum bending moment, the moment distribution curve for the pile was plotted as shown in Fig. 4. From this, the maximum moment for 20 feet of penetration is found to be 3.4 x 10^4 lb-ins. and occurs at a point 5.5 feet below the ground surface.

If the computed depth of maximum bending had turned out to be several feet different from the assumed value it would have been necessary to adjust the moment of inertia and repeat the computations. On the other hand, accuracy greater than about 1 to 2 feet is seldom justified.

Similar computations were made for different depths of penetration to produce the curves in Fig. 5. Note that there is little gain in lateral capacity for depths of penetration greater than about 40 feet. Since, in the example problem, the moment at the ground surface can be expressed as a linear function of the lateral load P, the curves in Fig. 5 can be directly proportioned for any allowable deflection at the mudline. Thus if 2 inches of deflection were allowed, the corresponding lateral load would be 500 lbs. and the maximum bending moment would be 6.8 x 10^4 lbs-ins. There also would be a very slight increase in the depth to the point of maximum bending.

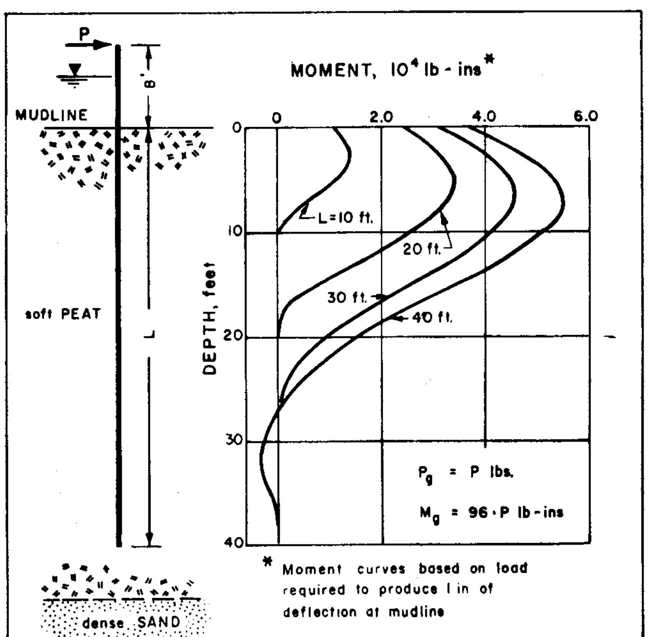


FIG. 4 TYPICAL LATERAL LOAD ANALYSIS SHOWING DISTRIBUTION OF BENDING MOMENT ALONG EMBEDDED PORTION OF PILE

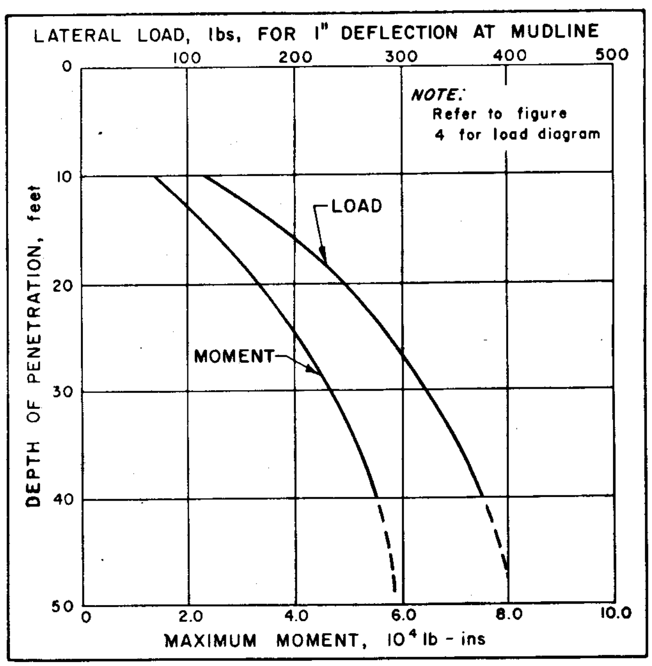


FIG. 5 LATERAL CAPACITY OF CLASS B TIMBER PILES EMBEDDED IN SOFT PEAT

Determination of the Depth to Fixity

In the preceding example, the resultant lateral load was applied at a point 8 feet above the ground surface. Since a free end condition was assumed at the top of the pile, the determination of the shear and moment at the mudline was a matter of simple statics. Often, with partially embedded piles, the top of the pile is fixed to some degree and the structure is then statically indeterminate. It is most convenient to the structural engineer if the pile shown in Fig. 6a can be replaced for the purpose of analysis, by an equivalent free standing pile (Fig. 6b) which is fixed at some depth, L_f, below the ground surface. Davisson and Robinson (7) have presented a theoretically correct solution for determining the depth to fixity, L_f, for long piles, i.e., $Z_{max} > 4$. Their solution satisfies the conditions that the deflection and rotation at the top of the equivalent pile, as well as the critical buckling load is the same as for the real pile.

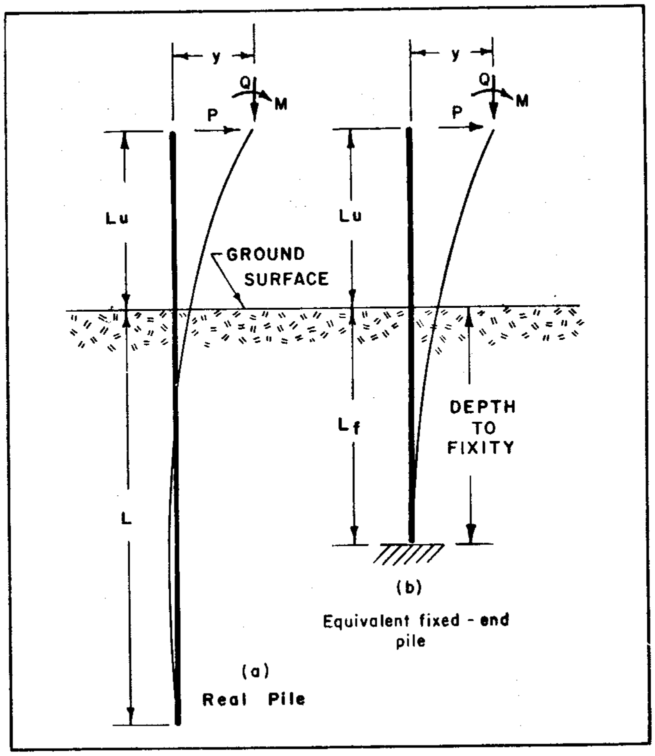


FIG. 6 DEFINITION OF DEPTH TO FIXITY

The depth to fixity is dependent upon the stiffness of the pile and the magnitude and variation of the soil resistance but is reasonably constant when expressed in terms of dimensionless parameters derived from the solution of an equation similar to equation 1. Davisson and Robinson found that when the ratio of the unsupported length, L_u (Fig. 6) to the relative stiffness factor (Table 1) exceeds 2 (k = constant) or unity ($k = n_h . x$), then L_f can be determined with little approximation from equations 4 and 5. Thus,

$$\text{If } k = \text{constant and } \frac{L_u}{R} > 2 \text{ then } L_f = 1.4\,R \qquad \ldots 4$$

$$\text{If } k = n_h . x \text{ and } \frac{L_u}{T} > 1 \text{ then } L_f = 1.8\,T \qquad \ldots 5$$

The equivalent beam defined by equations 4 and 5 can be used in conventional frame analyses for determining moments and loads at the top of the pile and for determining the buckling load for the pile. However, the moment computed for the fixed end of the equivalent pile will be considerably larger than the actual moment in the real pile. Therefore to analyze the embedded portion of the pile it is necessary to resort to the procedures previously discussed, using the moments and loads at the mudline. These can be determined from basic principles of statics once the conditions at the top of the pile have been determined from the frame analysis.

To illustrate the above point, refer to the example problem previously given. For 30 feet of embedment the ratio $L_u/T = (8 \times 12)/83.3 = 1.15$ and $Z_{max} = (30 \times 12)\ (83.3) = 4.3$. Therefore, the depth to fixity, $L_f = 1.8\ (83.3) = 150$ ins. The moment at this depth equals $(323)\ (150 + 96) = 4.84 \times 10^4$ lb-ins. By comparison, the moment at a depth of 150 ins., computed on the basis of elastic theory (Fig. 4) is about 3.5×10^4 lb-ins.

Conclusions

Theoretical solutions for determining the lateral capacity of piles are available. However, the selection of the design parameters and in particular, the properties of the supporting soil, must be tempered by experience and good engineering judgment. It is probable that with an increase in the use of the lateral pile load test a wealth of data eventually will become available which will modify and improve existing methods and perhaps encourage new ones. ■

REFERENCES

1. HETENYI, M., "Beams on Elastic Foundations," University of Michigan Press, Ann Arbor, 1946.
2. MATLOCK, H. and L. C. REESE, "Generalized Solutions for Laterally Loaded Piles," ASCE Journal, Soil Mechanics and Foundation Division, Vol. 86, No. SM 5, October 1960.
3. REESE, L. C. and H. MATLOCK, "Non-Dimensional Solutions for Laterally Loaded Piles with Soil Modulus Assumed Proportional to Depth," 8th Texas Conference on Soil Mechanics and Foundation Engineering, 1956.
4. DAVISSON, M. T. and H. L. GILL, "Laterally Loaded Piles in a Layered Soil System," ASCE Journal, Soil Mechanics and Foundation Division, Vol. 89, No. SM 3, May 1963.
5. DAVISSON, M. T. and S. PRAKASH, "A Review of Soil-Pole Behavior," HRB Record No. 39, p. 25, 1963.
6. TERZAGHI, K., "Evauation of Coefficients of Subgrade Reaction," Geotechnique, Vol. 5, p. 297, 1955.
7. DAVISSON, M. T. and K. E. ROBINSON, "Bending and Buckling of Partially Embedded Piles," 6th Inter. Conf. on Soil Mechanics and Foundation Engineering, Vol. 2, p. 243, 1965.
8. PRAKASH, S., "Behavior of Pile Groups Subjected to Lateral Loads," Ph.D. thesis, University of Illinois, 1962.
9. ROWE, P. W., Discussion of "Evaluation of Coefficients of Subgrade Reaction," by K. Terzaghi, Geotechnique, Vol. 6, p. 94, 1956.
10. PECK, R. B. and M. T. DAVISSON, Discussion of "Design and Stability Considerations for Unique Pier" by J. Michalos and D. P. Billington, ASCE Transactions, Vol. 127, Part IV, p. 413, 1962.